SOUTH CAROLINA ACADEMIC STANDARDS FOR MATHEMATICS

Algebra I

South Carolina Department of Education
Columbia, South Carolina

2007

Final State Board and Education Oversight Committee Approved Version
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State Mathematics Review Panel
District superintendents recommended educators from around the state to serve as members of the State Mathematics Review Panel. The panel reviewed and recommended revisions to the 2000 standards document, *South Carolina Mathematics Curriculum Standards.*

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Introduction

This document contains the revised academic standards in mathematics for South Carolina students from kindergarten through the twelfth grade. A field review of the first draft of these standards was conducted from October 10 through November 29, 2006. Feedback from that review was incorporated into the final draft, which was presented to the State Board of Education in January 2007.

The *South Carolina Mathematics Academic Standards* is not a curriculum. The academic standards in this document are not sequenced for instruction; do not prescribe classroom activities or materials; and do not dictate instructional strategies, approaches, or practices. A mathematics standards support document, issued by the State Department of Education (SDE), will serve as a resource for districts in constructing district-level standards-based mathematics curricula. By constructing an individual district mathematics curriculum, each district may expand or add topics and organize course content to fit its particular students' needs.

Development and Review of the South Carolina Mathematics Academic Standards

Beginning in 2004, the term for the state-approved expectations for student learning and academic performance in South Carolina was changed from *curriculum standards* to *academic standards*. The SDE, in partnership with Mid-Continent Research for Education and Learning, developed the academic standards and indicators for mathematics by utilizing a number of resources. Central among them were the *South Carolina Mathematics Curriculum Standards 2000*; recommendations made by the State Mathematics Review Panel; and the EOC report containing recommendations from national experts, parents, and business leaders.

The mathematics standards set forth in the *South Carolina Mathematics Curriculum Standards 2000* document were aligned with the national standards published in 2000 by the National Council of Teachers of Mathematics (NCTM) in the document *Principles and Standards for School Mathematics* (available online at http://standards.nctm.org/document/index.htm). Those national standards have also served as a guide for this 2007 edition of the South Carolina academic standards for mathematics and the supporting indicators. The academic standards documents of a number of states as well as the following publications were also utilized:

- *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (Reston, VA: NCTM, 2006—available online at http://www.nctm.org/focalpoints/downloads.asp)

- *Mathematics Assessment and Exercise Specifications for the National Assessment of Educational Progress*, developed by the Council of Chief State School Officers, NAEP Mathematics Consensus Project (Washington, DC: National Assessment Governing Board, U.S. Department of Education, n.d.)

- *Mathematics Framework for the 2005 National Assessment of Educational Progress*, developed by the Council of Chief State School Officers, NAEP Mathematics Project (Washington, DC: National Assessment Governing Board, U.S. Department of Education, n.d.)

Procedures for the review of all newly revised South Carolina academic standards, which were agreed upon by the SDE and the EOC, are published in the document *Procedures for the Cyclical Review of Current South Carolina K–12 Academic Standards and for the Development*

of New Academic Standards. Those procedures were used in the field review of the first draft of the revised standards document.

Changes in the South Carolina Mathematics Academic Standards Document

The structure and organization of the South Carolina mathematics standards document have been changed in several ways:

- Academic standards are specified for nine grade levels (kindergarten through grade eight) and for five high school core areas (elementary algebra, intermediate algebra, geometry, precalculus, and data analysis and probability).

- Each grade-level and core-area set of standards is now preceded by an overview page. For kindergarten through grade eight, the overview sets forth highlights of new learning. For the high school core areas, the overview provides information concerning the content of the standards with regard to the particular courses that are based on them. The mathematics standards and their indicators describe a connected body of mathematical processes, understandings, and competencies and should serve as the basis for the development of district-level curricula.

- The number of standards—which now ranges from six to seven for each grade or high school core area—has been significantly reduced.

- In kindergarten through grade eight, a standard is written specifically for each of the five mathematical strands—number and operations, algebra, geometry, measurement, and data analysis and probability—to afford a clear vertical articulation of content.

- Each grade level and high school core area begins with the mathematical processes standard, which centers in the specific methods that students will use in applying the skills and knowledge reflected in each five strands that follow this first standard: problem solving, reasoning and proof, communication, connections, and representation.

- The statements of the academic standards are newly constructed. Each of the standards that follow the mathematical processes standard is now stated as one full sentence that begins with the clause "The student will demonstrate through the mathematical processes . . ." and then goes on to specify particular content and skills. The verb "will demonstrate" is used with its general, everyday meaning and does not describe a cognitive category from the taxonomy.

  Following each of the academic standards are indicators, which are intended to help meet the need for grade-level specificity. The indicators are statements of the specific cognitive processes (expressed in the main verbs) and the content knowledge and skills that students must demonstrate in order to meet the grade-level or high school core-area standard.

  The main verbs in the indicators are taxonomic. That is, the main verbs identify specific aspects of the cognitive process as described in the revised Bloom's taxonomy (included in this standards document as the appendix). Use of the revised taxonomy will allow teachers to identify the kind of content (knowledge) addressed in the indicators (factual, conceptual, procedural, or metacognitive). In addition, use of the revised taxonomy will help teachers align lessons with both the content and the cognitive process identified in the indicators.

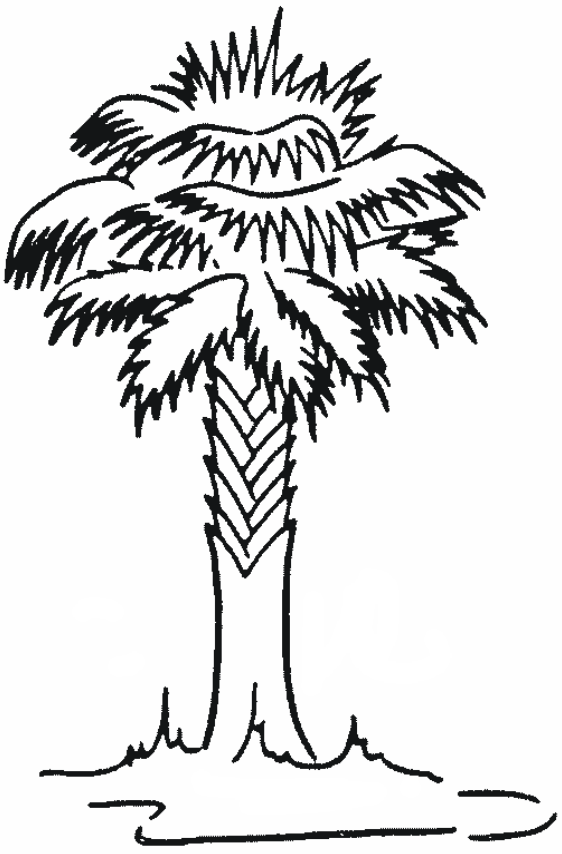
Many of the indicators in mathematics address conceptual knowledge and fall under the second category of cognitive processing, *understanding*, which fosters transfer and meaningful learning rather than rote learning and memorization. These revised mathematics standards also contain some indicators that require students to *analyze* or *evaluate* mathematical representations or situations. As a result, students must use understanding as they demonstrate even more cognitively complex learning.

Statewide Assessments

The mathematics standards and indicators for grades three through eight will be used as the basis for the Palmetto Achievement Challenge Tests (PACT) in mathematics. The mathematics standards for the high school core area of elementary algebra will be used as the basis for items on the state-required end-of-course examination for Algebra 1 and Mathematics for the Technologies 2.

The PACT is based on the broad standards at each grade level. Individual test questions will be aligned with the indicators and in most cases will measure the specific cognitive process stated in the main verb in the indicator. However, some indicators may be assessed through items that address other appropriate cognitive processes within the same category as the main verb in the indicator or may address processes in categories of lower cognitive complexity. For example, the assessment of an indicator that requires students to classify two-dimensional shapes as polygons or nonpolygons—which would fall in the second cognitive category, *understand*—might also ask the student to demonstrate other related cognitive processes such as comparing polygons and nonpolygons or giving examples of polygons or nonpolygons.

HIGH SCHOOL CORE-AREA STANDARDS



Elementary Algebra

Overview

The academic standards for the elementary algebra core area establish the process skills and core content for Algebra 1, Mathematics for the Technologies 1, and Mathematics for the Technologies 2, which should provide students with the mathematics skills and conceptual understanding necessary for them to further their mathematical education or to pursue mathematics-related technical careers. These standards will be the basis for the development of the items on the state-required end-of-course examination for Algebra 1 and Mathematics for the Technologies 2.

The content of the elementary algebra standards encompasses the real number system; operations involving exponents, matrices, and algebraic expressions; relations and functions; writing and solving linear equations; graphs and characteristics of linear equations; and quadratic relationships and functions. Teachers, schools, and districts should use the elementary algebra standards to make decisions concerning the structure and content of Algebra 1, Mathematics for the Technologies 1, and Mathematics for the Technologies 2. Content in these three courses may go beyond the elementary algebra standards.

All courses based on the academic standards for elementary algebra must include instruction using the mathematics process standards, allowing students to engage in problem solving, decision making, critical thinking, and applied learning. Educators must determine the extent to which such courses or individual classes may go beyond these standards. Such decisions will involve choices regarding additional content, activities, and learning strategies and will depend on the objectives of the particular courses or individual classes.

In all courses based on the elementary algebra standards, hand-held graphing calculators are required for instruction and assessment. Students should learn to use a variety of ways to represent data, to use a variety of mathematical tools such as graph paper, and to use technologies such as graphing calculators to solve problems.

Note: The term *including* appears in parenthetical statements in the high school mathematics indicators to introduce a list of specifics that are intended to clarify and focus the teaching and learning of the particular concept. That is, within these parenthetical including statements are specified the components of the indicator that are critical for the particular core area with regard both to the state assessments and to the management of time in the classroom. While instruction must focus on the entire indicator, educators must be certain to cover the components specified in the parenthetical *including* statements.

Elementary Algebra

The mathematical processes provide the framework for teaching, learning, and assessing in all high school mathematics core courses. Instructional programs should be built around these processes.

Standard EA-1: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

Indicators

EA-1.1 Communicate a knowledge of algebraic relationships by using mathematical terminology appropriately.
EA-1.2 Connect algebra with other branches of mathematics.
EA-1.3 Apply algebraic methods to solve problems in real-world contexts.
EA-1.4 Judge the reasonableness of mathematical solutions.
EA-1.5 Demonstrate an understanding of algebraic relationships by using a variety of representations (including verbal, graphic, numerical, and symbolic).
EA-1.6 Understand how algebraic relationships can be represented in concrete models, pictorial models, and diagrams.
EA-1.7 Understand how to represent algebraic relationships by using tools such as handheld computing devices, spreadsheets, and computer algebra systems (CASs).

Elementary Algebra

Standard EA-2: The student will demonstrate through the mathematical processes an understanding of the real number system and operations involving exponents, matrices, and algebraic expressions.

Indicators

EA-2.1 Exemplify elements of the real number system (including integers, rational numbers, and irrational numbers).

EA-2.2 Apply the laws of exponents and roots to solve problems.

EA-2.3 Carry out a procedure to perform operations (including multiplication and division) with numbers written in scientific notation.

EA-2.4 Use dimensional analysis to convert units of measure within a system.

EA-2.5 Carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions.

EA-2.6 Carry out a procedure to evaluate an expression by substituting a value for the variable.

EA-2.7 Carry out a procedure (including addition, subtraction, multiplication, and division by a monomial) to simplify polynomial expressions.

EA-2.8 Carry out a procedure to factor binomials, trinomials, and polynomials by using various techniques (including the greatest common factor, the difference between two squares, and quadratic trinomials).

EA-2.9 Carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication).

EA-2.10 Represent applied problems by using matrices.

Elementary Algebra

Standard EA-3: The student will demonstrate through the mathematical processes an understanding of relationships and functions.

Indicators

EA-3.1 Classify a relationship as being either a function or not a function when given data as a table, set of ordered pairs, or graph.

EA-3.2 Use function notation to represent functional relationships.

EA-3.3 Carry out a procedure to evaluate a function for a given element in the domain.

EA-3.4 Analyze the graph of a continuous function to determine the domain and range of the function.

EA-3.5 Carry out a procedure to graph parent functions (including $y = x$, $y = x^2$, $y = \sqrt{x}$, $y = |x|$, and $y = \frac{1}{x}$).

EA-3.6 Classify a variation as either direct or inverse.

EA-3.7 Carry out a procedure to solve literal equations for a specified variable.

EA-3.8 Apply proportional reasoning to solve problems.

Elementary Algebra

Standard EA-4: The student will demonstrate through the mathematical processes an understanding of the procedures for writing and solving linear equations and inequalities.

Indicators

EA-4.1 Carry out a procedure to write an equation of a line with a given slope and a *y*-intercept.
EA-4.2 Carry out a procedure to write an equation of a line with a given slope passing through a given point.
EA-4.3 Carry out a procedure to write an equation of a line passing through two given points.
EA-4.4 Use a procedure to write an equation of a trend line from a given scatterplot.
EA-4.5 Analyze a scatterplot to make predictions.
EA-4.6 Represent linear equations in multiple forms (including point-slope, slope-intercept, and standard).
EA-4.7 Carry out procedures to solve linear equations for one variable algebraically.
EA-4.8 Carry out procedures to solve linear inequalities for one variable algebraically and then to graph the solution.
EA-4.9 Carry out a procedure to solve systems of two linear equations graphically.
EA-4.10 Carry out a procedure to solve systems of two linear equations algebraically.

Elementary Algebra

Standard EA-5: The student will demonstrate through the mathematical processes an understanding of the graphs and characteristics of linear equations and inequalities.

Indicators

EA-5.1 Carry out a procedure to graph a line when given the equation of the line.
EA-5.2 Analyze the effects of changes in the slope, m, and the y-intercept, b, on the graph of $y = mx + b$.
EA-5.3 Carry out a procedure to graph the line with a given slope and a y-intercept.
EA-5.4 Carry out a procedure to graph the line with a given slope passing through a given point.
EA-5.5 Carry out a procedure to determine the x-intercept and y-intercept of lines from data given tabularly, graphically, symbolically, and verbally.
EA-5.6 Carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally.
EA-5.7 Apply the concept of slope as a rate of change to solve problems.
EA-5.8 Analyze the equations of two lines to determine whether the lines are perpendicular or parallel.
EA-5.9 Analyze given information to write a linear function that models a given problem situation.
EA-5.10 Analyze given information to determine the domain and range of a linear function in a problem situation.
EA-5.11 Analyze given information to write a system of linear equations that models a given problem situation.
EA-5.12 Analyze given information to write a linear inequality in one variable that models a given problem situation.

Elementary Algebra

Standard EA-6: The student will demonstrate through the mathematical processes an understanding of quadratic relationships and functions.

Indicators

EA-6.1 Analyze the effects of changing the leading coefficient *a* on the graph of $y = ax^2$.

EA-6.2 Analyze the effects of changing the constant *c* on the graph of $y = x^2 + c$.

EA-6.3 Analyze the graph of a quadratic function to determine its equation.

EA-6.4 Carry out a procedure to solve quadratic equations by factoring.

EA-6.5 Carry out a graphic procedure to approximate the solutions of quadratic equations.

EA-6.6 Analyze given information to determine the domain of a quadratic function in a problem situation.

Table 1: The Knowledge Dimension

Major Types and Subtypes		Examples
A. Factual Knowledge—The basic elements students must know to be acquainted with a discipline or solve problems in it		
Aa.	Knowledge of terminology	Technical vocabulary, musical symbols
Ab.	Knowledge of specific details and elements	Major natural resources, reliable sources of information
B. Conceptual Knowledge—The interrelationships among the basic elements within a larger structure that enable them to function together		
Ba.	Knowledge of classifications and categories	Periods of geological time, forms of business ownership
Bb.	Knowledge of principles and generalizations	Pythagorean theorem, law of supply and demand
Bc.	Knowledge of theories, models, and structures	Theory of evolution, structure of Congress
C. Procedural Knowledge—How to do something, methods and inquiry, and criteria for using skills, algorithms, techniques, and methods		
Ca.	Knowledge of subject-specific skills and algorithms	Skills used in painting with watercolors, whole-number division algorithm
Cb.	Knowledge of subject-specific techniques and methods	Interviewing techniques, scientific method
Cc.	Knowledge of criteria for determining when to use appropriate procedures	Criteria used to determine when to apply a procedure involving Newton's second law, criteria used to judge the feasibility of using a particular method to estimate business costs
D. Metacognitive Knowledge—Knowledge of cognition in general as well as awareness and knowledge of one's own cognition		
Da.	Strategic knowledge	Knowledge of outlining as a means of capturing the structure of a unit of subject matter in a textbook, knowledge of the use of heuristics
Db.	Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge	Knowledge of the types of tests particular teachers administer, knowledge of the cognitive demands of different tasks
Dc.	Self-knowledge	Knowledge that critiquing essays is a personal strength, whereas writing essays is a personal weakness; awareness of one's own knowledge level

Table 2: The Cognitive Process Dimension

Categories & Cognitive Processes	Alternative Names	Definitions and Examples
1. REMEMBER—Retrieve relevant knowledge from long-term memory		
1.1 Recognizing	Identifying	Locating knowledge in long-term memory that is consistent with presented material (e.g., Recognize the dates of important events in United States history)
1.2 Recalling	Retrieving	Retrieving relevant knowledge from long-term memory (e.g., Recall the dates of important events in United States history)
2. UNDERSTAND—Construct meaning from instructional messages, including oral, written, and graphic communication		
2.1 Interpreting	Clarifying, paraphrasing, representing, translating	Changing from one form of representation (e.g., numerical) to another (e.g., verbal) (e.g., Paraphrase important speeches and documents)
2.2 Exemplifying	Illustrating, instantiating	Finding a specific example or illustration of a concept or principle (e.g., Give examples of various artistic painting styles)
2.3 Classifying	Categorizing, subsuming	Determining that something belongs to a category (e.g., Classify observed or described cases of mental disorders)
2.4 Summarizing	Abstracting, generalizing	Abstracting a general theme or major point(s) (e.g., Write a short summary of events portrayed on a videotape)
2.5 Inferring	Concluding, extrapolating, interpolating, predicting	Drawing a logical conclusion from presented information (e.g., In learning a foreign language, infer grammatical principles from examples)
2.6 Comparing	Contrasting, mapping, matching	Detecting correspondences between two ideas, objects, and the like (e.g., Compare historical events to contemporary situations)
2.7 Explaining	Constructing models	Constructing a cause-and-effect model of a system (e.g., Explain the causes of important 18th Century events in France)
3. APPLY—Carry out or use a procedure in a given situation		
3.1 Executing	Carrying out	Applying a procedure to a familiar task (e.g., Divide one whole number by another whole number, both with multiple digits)
3.2 Implementing	Using	Applying a procedure to an unfamiliar task (e.g., Use Newton's Second Law in situations in which it is appropriate)

Table 2: The Cognitive Process Dimension

Categories & Cognitive Processes	Alternative Names	Definitions and Examples
4. ANALYZE—Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose		
4.1 Differentiating	Discriminating, distinguishing, focusing, selecting	Distinguishing relevant from irrelevant parts or important from unimportant parts of presented material (e.g., Distinguish between relevant and irrelevant numbers in a mathematical word problem)
4.2 Organizing	Finding coherence, integrating, outlining, parsing, structuring	Determining how elements fit or function within a structure (e.g., Structure evidence in a historical description into evidence for and against a particular historical explanation)
4.3 Attributing	Deconstructing	Determine a point of view, bias, values, or intent underlying presented material (e.g., Determine the point of view of the author of an essay in terms of his or her political perspective)
5. EVALUATE—Make judgments based on criteria and standards		
5.1 Checking	Coordinating, detecting, monitoring, testing	Detecting inconsistencies or fallacies within a process or product; determining whether a process or product has internal consistency; detecting the effectiveness of a procedure as it is being implemented (e.g., Determine if a scientist's conclusions follow from observed data)
5.2 Critiquing	Judging	Detecting inconsistencies between a product and external criteria, determining whether a product has external consistency; detecting the appropriateness of a procedure for a given problem (e.g., Judge which of two methods is the best way to solve a given problem)
6. CREATE—Put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure		
6.1 Generating	Hypothesizing	Coming up with alternative hypotheses based on criteria (e.g., Generate hypotheses to account for an observed phenomenon)
6.2 Planning	Designing	Devising a procedure for accomplishing some task (e.g., Plan a research paper on a given historical topic)
6.3 Producing	Constructing	Inventing a product (e.g., Build habitats for a specific purpose)

Table 3: A Taxonomy for Teaching, Learning, and Assessing

THE KNOWLEDGE DIMENSION	THE COGNITIVE PROCESS DIMENSION					
	1. Remember—Retrieve relevant knowledge from long-term memory 1.1 Recognizing 1.2 Recalling	**2. Understand**—Construct meaning from instructional messages, including oral, written, and graphic communication 2.1 Interpreting 2.2 Exemplifying 2.3 Classifying 2.4 Summarizing 2.5 Inferring 2.6 Comparing 2.7 Explaining	**3. Apply**—Carry out or use a procedure in a given situation 3.1 Executing 3.2 Implementing	**4. Analyze**—Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose 4.1 Differentiating 4.2 Organizing 4.3 Attributing	**5. Evaluate**—Make judgments based on criteria and standards 5.1 Checking 5.2 Critiquing	**6. Create**—Put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure 6.1 Generating 6.2 Planning 6.3 Producing
A. Factual Knowledge—The basic elements that students must know to be acquainted with a discipline or solve problems in it AA. Knowledge of terminology AB. Knowledge of specific details and elements						
B. Conceptual Knowledge—The interrelationships among the basic elements within a larger structure that enable them to function together BA. Knowledge of classifications and categories BB. Knowledge of principles and generalizations BC. Knowledge of theories, models, and structures						
C. Procedural Knowledge—How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques, and methods CA. Knowledge of subject-specific skills and algorithms CB. Knowledge of subject-specific techniques and methods CC. Knowledge of criteria for determining when to use appropriate procedures						
D. Metacognitive Knowledge—Knowledge of cognition in general as well as awareness of one's own cognition DA. Strategic knowledge DB. Knowledge about cognitive tasks (including appropriate contextual and conditional knowledge DC. Self-knowledge						